MATH 1A - MOCK FINAL DELUXE

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Name: ___________________

Instructions: This is a mock final deluxe, designed to give you an idea of what the actual final deluxe will look like.

Careful! The actual final deluxe might have very different questions!

1		10
2		10
3		10
4		15
5		20
6		15
7		20
8		30
9		10
10		10
Bonus 1		5
Bonus 2		5
Bonus 3		5
Total		150

Date: Friday, August 5th, 2011.

1. *(10 points, 5 points each)* Find the following limits

(a) $\lim_{x\to\infty} \sqrt{x^2+x} - x$

(b) $\lim_{x\to\infty} \frac{(\ln(x))^2}{x}$

2. *(10 points)* Use the **definition** of the derivative to calculate $f'(x)$, where:

$$f(x) = \frac{1}{x}$$

3. *(10 points, 5 points each)* Find the derivatives of the following functions
   (a) $f(x) = x^{\cos(x)}$

   (b) y', where $x^3 + y^3 = xy$

4. *(15 points)* A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder is sliding away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

5. *(20 points)* If $12\pi\ cm^2$ of material is available to make a cylinder with an open top, find the largest possible volume of the cylinder.

6. *(15 points)* Show that the following equation has exactly one solution in [-1,1]

$$x^4 - 5x + 1 = 0$$

7. *(20 points)* Use the **definition** of the integral to find:

$$\int_1^2 x^2 dx$$

You may use the following formulas:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

8. *(30 points, 5 points each)* Find the following:

(a) The antiderivative F of $f(x) = x^2 + 3x^3 - 4x^7$ which satisfies $F(0) = 1$

(b) $\int_{-1}^{1} |x| \, dx$ (**Hint:** Draw a picture)

(c) $\int x^2 + 1 + \frac{1}{x^2+1} dx$

(d) $\int_1^e \frac{(\ln(x))^2}{x} dx$

(e) $g'(x)$, where $g(x) = \int_{x}^{e^x} \sqrt{1+t^2} dt$

(f) The average value of $f(x) = \sin(x)$ on $[-\pi, \pi]$

9. *(10 points)* Find the area of the region enclosed by the curves:

$$y = x^2 - 4 \quad \text{and} \quad y = 4 - x^2$$

10. *(10 points)* If $f(x) = x\ln(x)$, find:

(a) Intervals of increase and decrease, and local max/min

(b) Intervals of concavity and inflection points

Bonus 1 *(5 points)* Show that if f is continuous on $[0, 1]$, then $\int_0^1 f(x)dx$ is bounded, that is, there are numbers m and M such that:

$$m \leq \int_0^1 f(x)dx \leq M$$

Hint: Use one of the 'value' theorems we haven't used much in this course (see section 4.1)

Bonus 2 *(5 points)* If $f(x) = Ax^3 + Bx^2 + Cx + D$ is a polynomial whose coefficients satisfy:

$$\frac{A}{4} + \frac{B}{3} + \frac{C}{2} + D = 0$$

Show that f has at least one zero on $[0, 1]$.

Hint: What is the *average* value of f on $[0, 1]$?

Bonus 3 *(5 points)* Another way to define $\ln(x)$ is:

$$\ln(x) = \int_1^x \frac{1}{t} dt$$

Show **using this definition only** that $\ln(ab) = \ln(a) + \ln(b)$.

Hint: Fix a constant a, and consider the function:

$$f(x) = \ln(ax) - \ln(x) - \ln(a)$$